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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- A Single Axis, Two Maneuver
Gravity-Gradient Dump Procedure
for AAP-ATM Missions

TM- 68-1022-4

DATE- September 20, 1968

FILING CASE NO(S)- 620

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FILING SUBJECT(S)-
(ASSIGNED BY AUTHOR(S)- Apollo Applications
Attitude Control
Control Moment Gyros
Gravity-Gradient Momentum Dump

ABSTRACT

A single rotation, two maneuver gravity-gradient momentum dump procedure is shown to be feasible for the CM-SM/LM-ATM decoupled vehicle and as a manual backup for the orbital workshop cluster. More general schemes exist which can handle the bias momentum due to other disturbing torques as well as gravity-gradient. However, this single rotation scheme is simple in implementation for the decoupled vehicle and in crew backup procedures for the cluster. Maneuvering is done during orbital darkness to allow full use of orbital daylight for solar experiments.

When the angle between the orbital plane and the earth-sun line is less than approximately 30° the entire daylight accumulation of momentum produced by gravity-gradient torque can be dumped during orbital darkness. For angles greater than 30°, reaction thrust dumping is also required.

Depending upon launch date, a 28 day decoupled mission in circular orbit requires up to 340 lbs of propellant if only reaction thrust dumping is used. When supplemented by gravity-gradient dumping, 75 lbs are required.

Similarly, gravity-gradient dumping reduces the dump propellant requirement for a 56 day cluster mission from 286 lbs to 20 lbs.

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FROM: W. Levidow

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TECHNICAL MEMORANDUM

INTRODUCTION

The need to plan for an AAP ATM decoupled mission,
and the need for a simple manual method[1] of gravity-gradient
(g.g.) momentum dumping of the AAP-3/AAP-4 cluster have
suggested a look into single axis g.g. dump maneuvers.

As shown in Figure 1 below, during solar viewing the
decoupled CM-SM/LM-ATM will fly with its long axis rotated an
angle β out of the orbital plane, where β is the angle between
the plane and the earth-sun line. In contrast, the cluster
will fly with its long axis in the plane, its telescope axis
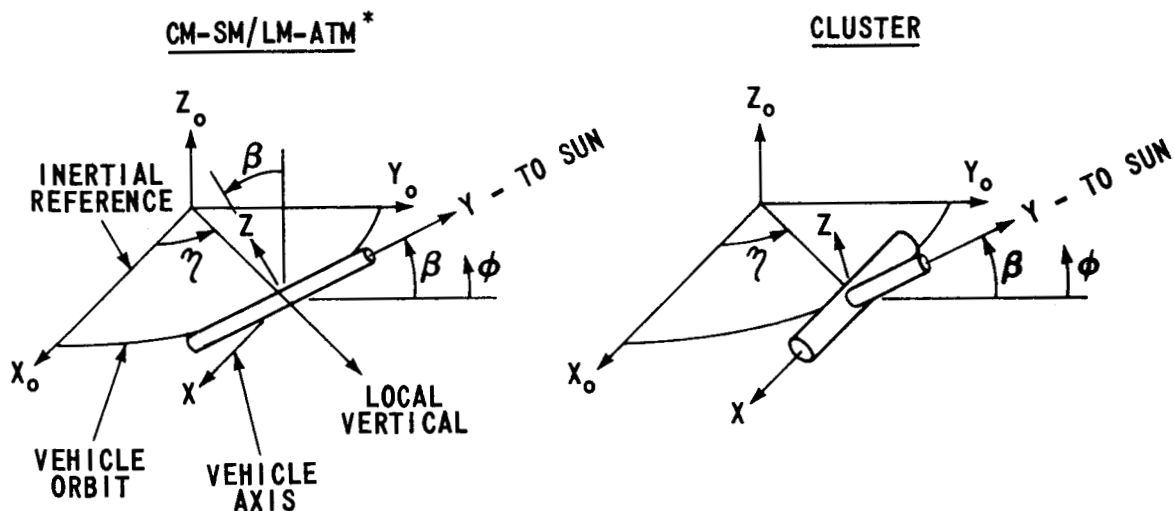being rotated β out of plane toward the sun.

FIGURE I

*The CM-SM/LM-ATM vehicle axes have been chosen in this memorandum
for convenience of analysis; they do not conform to the usual definition
of the axes for this vehicle.

The g.g. and aerodynamic torques are counteracted during attitude hold by the momentum exchange action of the Control Moment Gyros (CMGs) mounted on the ATM rack. However, the CMGs have a limited capacity to absorb momentum. For continued CMG attitude control, accumulations of momentum due to bias components of the torques must be dumped periodically before momentum saturation is reached. This dumping action may be realized by maneuvering the vehicle to a dump attitude such that the g.g. torques act to restore the momentum to a desired level.

This memorandum will define the g.g.* momentum accumulated during solar viewing, determine the g.g. dump capability during orbital darkness, suggest a method of momentum management, and determine for the proposed missions the weight of reaction thrust propellant required to supplement g.g. dumping.

CONCEPT OF SINGLE AXIS GRAVITY-GRADIENT DUMP

The g.g. torques acting on the vehicle can be expressed in terms of the attitude of the vehicle principal axes relative to an inertial coordinate system. The inertial system chosen (Figure 1) is such that the Z_0 axis is normal to the orbital plane, positive toward north, and the X_0 axis points in the direction of the vector formed by crossing the earth-sun vector into the Z_0 axis vector. The Y_0 axis completes a right handed orthogonal system.

The components of the g.g. torques about the principal axes of a vehicle in circular orbit are:

$$T_X = \frac{3}{2} \omega^2 [\sin 2\phi \sin^2\eta (I_Y - I_Z)] \text{ ft-lbs}$$

$$T_Y = \frac{3}{2} \omega^2 [\sin \phi \sin 2\eta (I_Z - I_X)] \text{ ft-lbs}$$

$$T_Z = \frac{3}{2} \omega^2 [-\cos \phi \sin 2\eta (I_X - I_Y)] \text{ ft-lbs}$$

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*Bias components of aerodynamic torque also produce momentum changes requiring dumping. These torques are smaller than the g.g. torques and their effect and remedy are now under study.

where ω = orbital rate, radians/sec.

ϕ = angular displacement of vehicle about the X_o axis.
($\phi=\beta$ during solar viewing)

η = angle between the local vertical and the X_o inertial reference axis

I_X, I_Y, I_Z = vehicle principal moments of inertia, slug-ft^2.

To illustrate these equations, a plot of g.g. torques vs. η for the decoupled vehicle is shown below in Figure 2 for $\phi=-45°$. Relative amplitudes depend upon the value of ϕ; $T_Y \approx 0$ because $I_X \approx I_Z$.
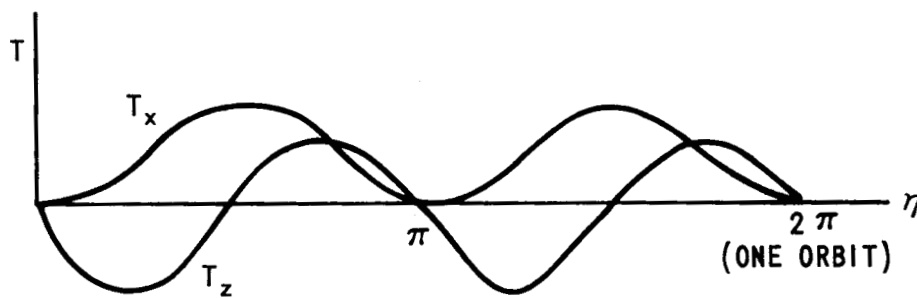
FIGURE 2 - GRAVITY GRADIENT TORQUES ON DECOUPLED VEHICLE

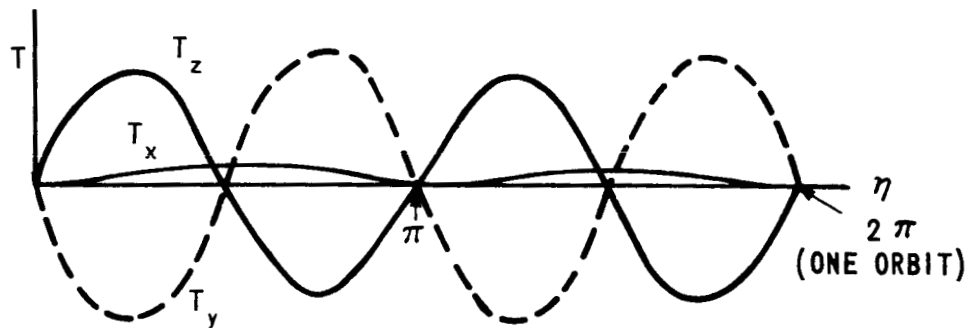To a different scale, the g.g. torques on the cluster for $\phi=-45°$ are shown below in Figure 3.

FIGURE 3 - GRAVITY GRADIENT TORQUES ON CLUSTER

Notice that for both vehicles the Y and Z torques are cyclic. However, the X axis torques exhibit a bias which will produce a unidirectional CMG momentum change. By inspection of the torque equations we see that if the vehicles are maneuvered to an attitude such that sin 2ϕ changes sign, the bias torques reverse direction and momentum dumping occurs.

To avoid interference with the solar experiments, we assume that the vehicles are maneuvered to the dump attitude only during orbital darkness. As will be shown, the limited orbital darkness interval and the time required for executing the maneuver will often prevent complete dumping of the momentum accumulated during solar viewing.

CMG MOMENTUM CHANGES

During CMG attitude control, the torque exerted by the vehicle on the CMG is equal to and in the same direction as the torque exerted by gravity on the vehicle. The CMG momentum change along a vehicle axis during any time interval can be obtained by integrating the torque equations over that interval.

For an interval from $t_1 = \dfrac{\eta_1}{\omega}$ to $t_2 = \dfrac{\eta_2}{\omega}$,

$$\Delta H = \int_{\frac{\eta_1}{\omega}}^{\frac{\eta_2}{\omega}} T(t)dt = \frac{1}{\omega} \int_{\eta_1}^{\eta_2} T(\eta)d\eta$$

resulting in:

$$\Delta H_X = \frac{3}{4}\omega \left[\sin 2\phi(\eta_2-\eta_1- \tfrac{1}{2}(\sin 2\eta_2-\sin 2\eta_1))(I_Y - I_Z) \right] \text{ ft-lb-sec}$$

$$\Delta H_Y = \frac{3}{4}\omega \left[-\sin \phi \, (\cos 2\eta_2-\cos 2\eta_1)(I_Z - I_X) \right] \text{ ft-lb-sec}$$

$$\Delta H_Z = \frac{3}{4}\omega \left[\cos \phi \, (\cos 2\eta_2-\cos 2\eta_1)(I_X - I_Y) \right] \text{ ft-lb-sec}$$

The angular intervals of solar pointing and dumping will be called θ_P and θ_D. Normally these intervals will be symmetrical about the earth-sun line, as shown below in Figure 4.
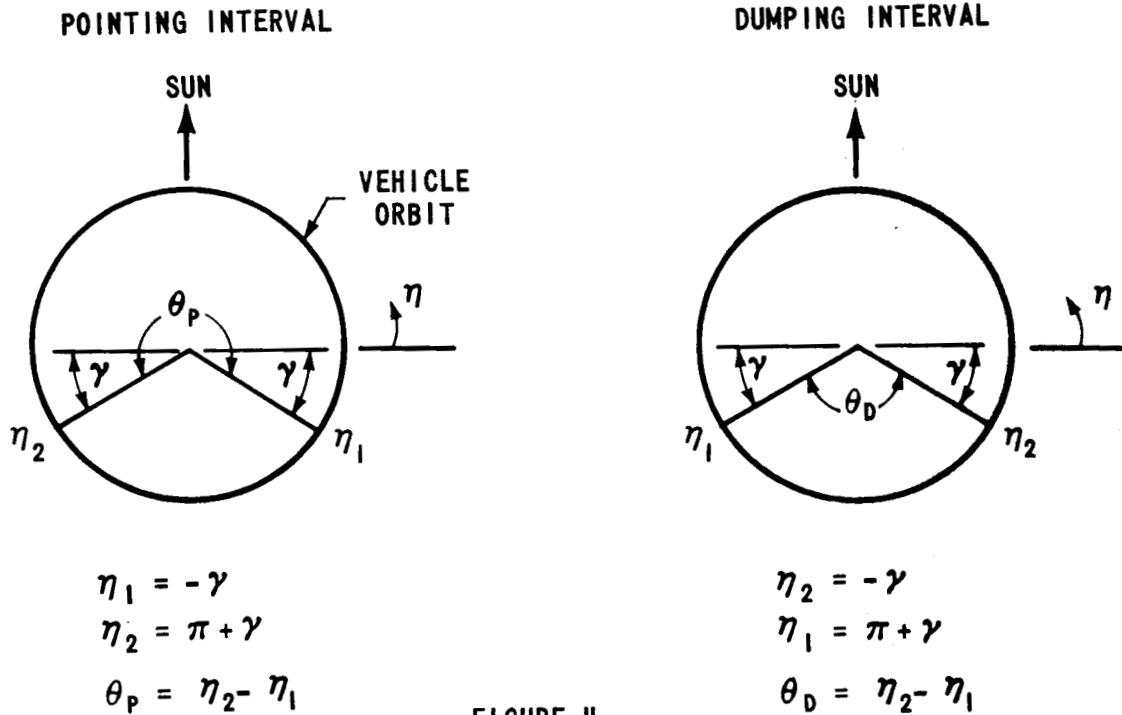
POINTING INTERVAL   **DUMPING INTERVAL**

$$\eta_1 = -\gamma$$
$$\eta_2 = \pi + \gamma$$
$$\theta_P = \eta_2 - \eta_1$$

$$\eta_2 = -\gamma$$
$$\eta_1 = \pi + \gamma$$
$$\theta_D = \eta_2 - \eta_1$$

FIGURE 4

For symmetrical pointing and dumping intervals, the ΔH equations become:

$$\Delta H_X = \frac{3}{4}\frac{\omega}{} \sin 2\phi \ (n_2 - n_1 + \sin 2n_1)(I_Y - I_Z)$$

$$\Delta H_Y = 0$$

$$\Delta H_Z = 0$$

Hence over the pointing and dumping intervals, the Y and Z axis momentum changes are zero and the X axis momentum changes are:

$$\Delta H_P = \frac{3}{4}\frac{\omega}{} \sin 2\beta \quad (\theta_P + \sin \theta_P)(I_Y - I_Z) \text{ ft-lb-sec.}$$

$$\Delta H_D = \frac{3}{4}\frac{\omega}{} \sin 2\phi_D \quad (\theta_D + \sin \theta_D)(I_Y - I_Z)$$

where β and ϕ_D are the values of ϕ during solar pointing and dumping.

For complete dumping, we require that $\Delta H_D = -\Delta H_P$. Hence $\sin 2\phi_D$ must be of opposite sign from $\sin 2\beta$. Rotating the vehicle to and from the appropriate dump attitude results in a single-axis, two-maneuver dump procedure.

Maximum dumping capability occurs when $\phi_D = \pm 45°$ and $\pm 135°$. The maximum dump capability is then

$$\Delta H_{Dm} = \pm \frac{3}{4}\frac{\omega}{} \quad (\theta_D + \sin \theta_D)(I_Y - I_Z)$$

For a given β there is a choice of two values of ϕ_D, 180° apart, to obtain maximum dumping. There is also a choice of direction of vehicle rotation in maneuvering to and from the chosen dump attitude. Selection of dump attitude and rotational direction depend on the required maneuver angle and the CMG momentum available for the maneuver, for both affect the maneuver time.

When the solar pointing momentum change exceeds the g.g. dump capability, the excess can be dumped before reaching saturation by use of reaction thrust. Such dumping may be scheduled once per orbit or less frequently as required.

Assuming maximum utilization of orbital daylight for solar pointing, the pointing interval θ_P coincides with the orbital daylight interval. The pointing interval is then defined by (2)

$$\cos \theta_P = \frac{-\cos \left[\sin^{-1} \left(\frac{R}{R + OH}\right)\right]}{\cos \beta}$$

where OH = orbital height

R = effective earth radius including atmosphere

For purposes of this study, a 230 NM circular orbit was chosen as representative of missions for both vehicles. Figure 5 shows the variation with β of the daylight pointing interval θ_P* and the g.g. dump characteristics of the CM-SM/ LM-ATM in such an orbit. The effect of maneuver time is neglected. For $|\beta| < 35°$ all the momentum accumulated during daylight pointing can be dumped during orbital darkness. For $|\beta| > 35°$, the pointing momentum exceeds the dump capability. The undumped momentum for this orbit reaches a maximum at $|\beta| = 69.6°$, at which time the complete orbit lies in daylight.

Table 1 presents the corresponding data for both vehicles. H_u is the undumped momentum.

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*In this memorandum the effect of earth atmosphere is not considered. For certain experiments the atmosphere decreases the useful solar pointing interval, thus increasing the dump capability.
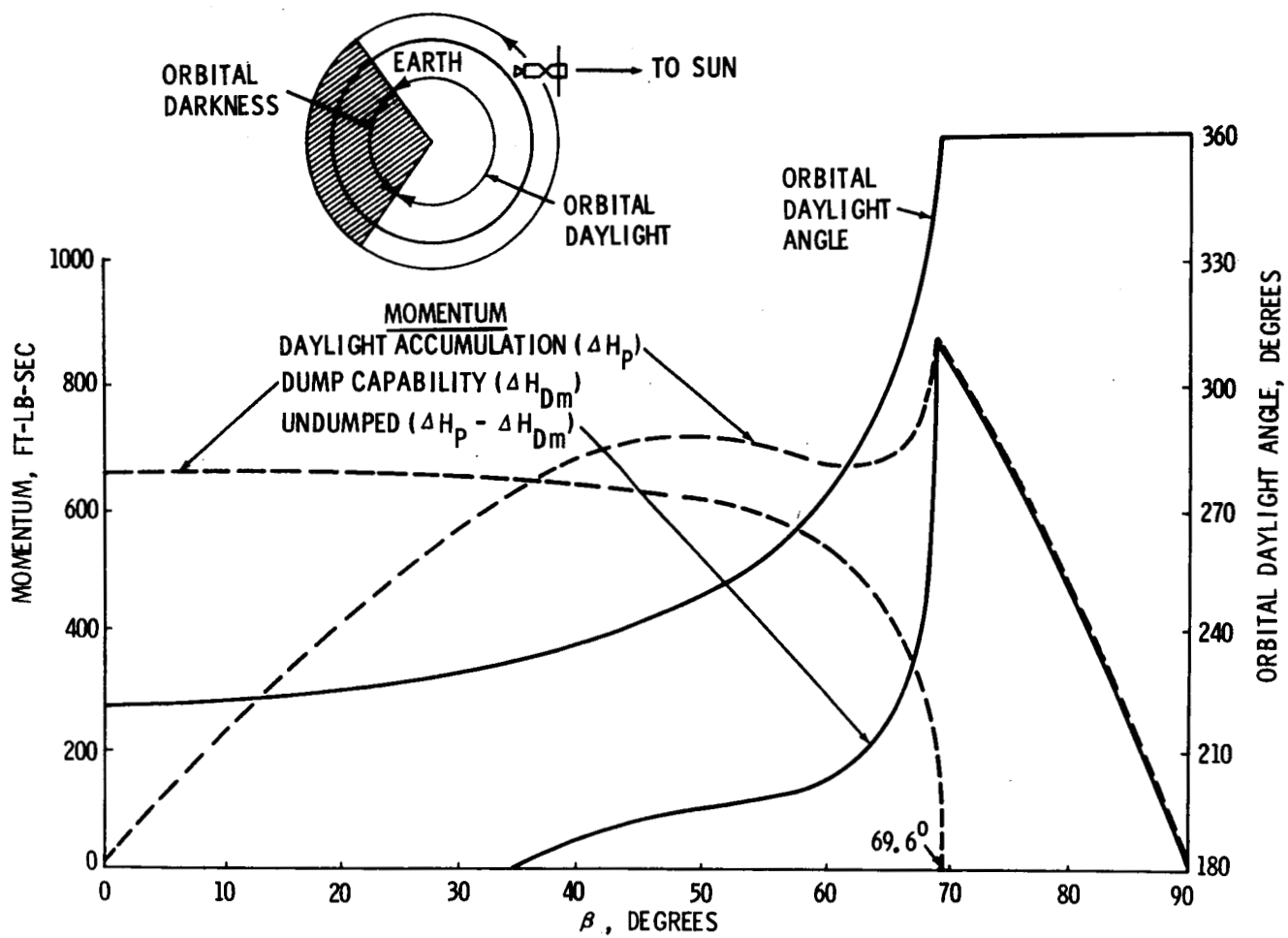
FIGURE 5 - CM-SM/LM-ATM GRAVITY GRADIENT DUMP CHARACTERISTICS

Table 1 - X Axis Momentum Changes Per Orbit
(Assuming Negligible Maneuver time)

β	θ_P	Decoupled (CM-SM/LM-ATM)			Cluster		
		ΔH_P	ΔH_{Dm}	H_u	ΔH_P	ΔH_{Dm}	H_u
Deg.	Deg.	(ft.-lb.-sec.)			(ft.-lb.-sec.)		
0	221	0	-650	0	0	-278	0
-10	222	232	-648	0	99	-277	0
-20	224	436	-647	0	187	-277	0
-30	227	591	-643	0	253	-275	0
-40	234	680	-634	46	290	-270	20
-50	246	700	-612	88	300	-262	38
-60	268	672	-548	124	288	-234	54
-69.6	360	865	0	865	376	0	376
-70	360	852	0	852	365	0	365
-80	360	453	0	0	194	0	194
-90	360	0	0	0	0	0	0

		Decoupled	Cluster	
Note:	$I_X =$	4.14×10^5	0.583×10^6	slug-ft^2
	$I_Y =$	1.66×10^5	3.763×10^6	slug-ft^2
	$I_Z =$	4.14×10^5	3.870×10^6	slug-ft^2

MOMENTUM MANAGEMENT

        The CMG control system consists of three double
gimballed gyros, each with a spin angular momentum of
2000 ft-lb-sec. The resultant spin angular momentum vector
$\underline{H}$ can be varied in any direction from zero to 6000 ft-lb-sec
by proper orientation of the gyro spin axes. Control torques
on the vehicle are produced by controlling the rate of change
of $\underline{H}$.

        Figure 6 below shows a 6000 ft-lb-sec sphere centered
on the inertial axis and represents the limit of $|\underline{H}|$. Gimbal
stops prevent the attainment of the full 6000 along the vehicle
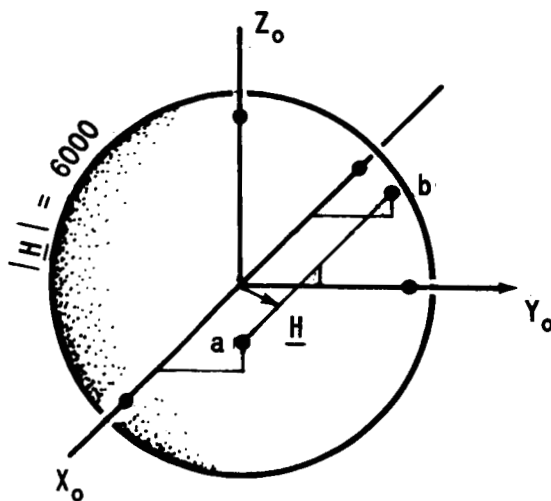axes, but the effect is not important for this discussion.

FIGURE 6

Positive X axis torque is produced by driving $\underline{H}$ in the negative X direction, such as along line a b. The change in angular velocity is proportional to the change, ΔH_X, of the X axis component of $\underline{H}$. Hence an X_o axis maneuver to a new attitude requires driving $\underline{H}$ in the X_o direction for acceleration and subsequently driving it back to its original value for deceleration.

It is evident that if $\underline{H} = 0$ prior to a maneuver, $\underline{H}$ can move out along the X axis to -6000 ($\Delta H_X = -6000$) for acceleration and then back to zero ($\Delta H_X = +6000$) for deceleration. However, if $\underline{H}$ has Y and Z components the H excursion parallel to the X axis is limited by the intersection with the $|\underline{H}| = 6000$ surface such as at a and b. We conclude therefore that for maximum X axis rotational velocities, the Y and Z components should be a minimum at maneuver time.

To develop a procedure for momentum management, let us consider the case of $\beta = -40°$ for the cluster and trace the variation of $\underline{H}$ for one orbit. Suppose at the center of the dump interval ($\eta = 270°$) the gyro spin axis are oriented such that $H_X = 0$ as shown below in Figure 7.
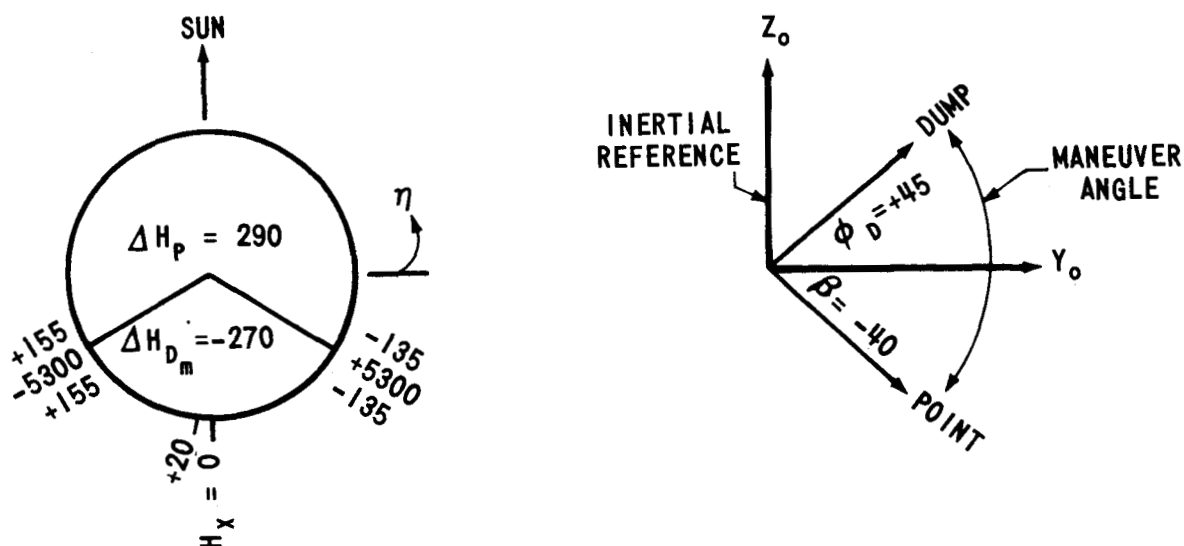

FIGURE 7 - CLUSTER, $\beta = -40°$

Assuming g.g. dumping at the maximum rate, upon entering daylight $H_X = \frac{-270}{2} = -135$ (see Table 1). A -X axis maneuver is then to be made through 85° to $\beta = -40°$.

It turns out that in traversing from $\eta = 270°$ to $\eta = 360°$ at a +45° dumping attitude, $\Delta H_Y \approx -4000$ and $\Delta H_Z \approx +4000$. It follows that if H_Y were set at +2000 and H_Z at -2000 when $\eta = 270°$, H_Y or H_Z would always have a magnitude less than 2000 at any maneuver initiated between $\eta = 270°$ and $\eta = 360°$. Assuming this maximum value of H_Y and H_Z for all maneuvers, the corresponding limits of H_X are ± 5300.

Hence for our maneuver from dump to pointing the gyro spin axes are moved such that H_X is driven from -135 to +5300 during rotation and back to -135 upon reaching the pointing attitude.

Upon reaching orbital darkness, $H_X = -135 + 290 = +155$. Due to symmetry of the pointing interval about the earth sun line, H_Y and H_Z equal their values at the previous maneuver. The CMGs now maneuver the vehicle back through 85° to the dump attitude, H_X moving to -5300 during rotation and then back to +155. Additional dumping occurs until upon reaching the starting point $H_X = +155 - \frac{270}{2} = 20$. This excess momentum may be dumped with reaction thrust at this point or accumulated for less frequent dumping.

For pointing angles of magnitude greater than 45°, such as $\beta = -50°$, the vehicle may be rotated 85° to a -135° dump attitude and then back to the pointing attitude as shown in Figure 8. H_X is driven to -5300 when maneuvering to pointing and to +5300 when maneuvering to dump.
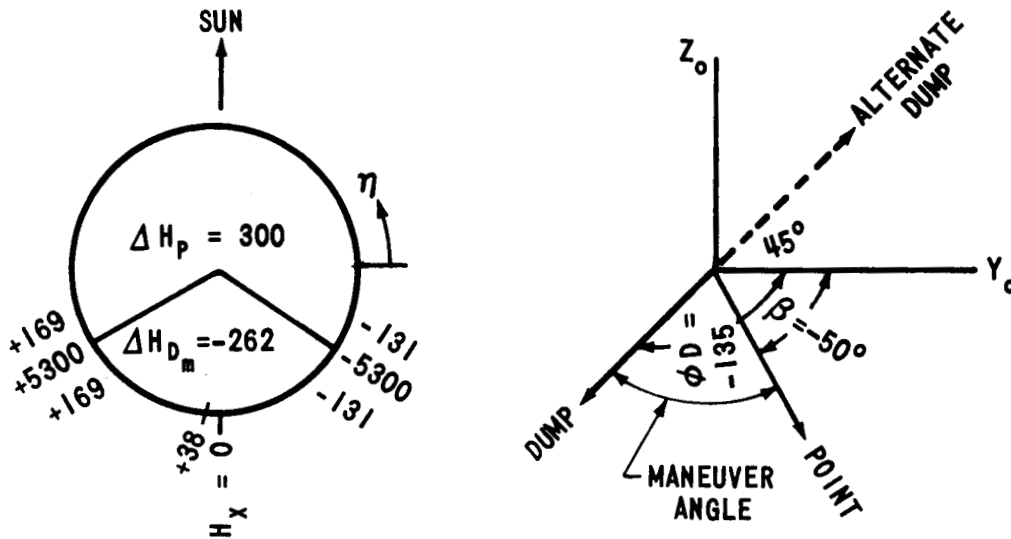
FIGURE 8 - CLUSTER, β = -50°

Notice that the ΔH_X available for the maneuver from pointing to dump is $(5300 + 155) = 5455$ for $\beta = -40°$ and $(5300 - 169) = 5131$ for $\beta = -50°$. For maneuver angles less than 90°, as in the above examples, the available ΔH_X will be greater than 5300 for $|\beta| < 45°$ and less than 5300 for $|\beta| > 45°$. The smaller ΔH_X will result in smaller vehicle rotational velocities. The velocity may be increased by rotating in the opposite direction to the alternate dump attitude 180° away, but the larger rotational angle will prevent a significant reduction in maneuver time.

MANEUVER TIME

From Table 1 we see that the minimum ΔH_X available for maneuvering the cluster equals $(5300 - 376) = 4924$ ft-lb-sec.

A similar analysis of the decoupled vehicle shows that if H_Y and H_Z are set equal to zero at $\eta = 270°$, their

magnitudes during maneuvers will be less than 300. This will have a negligible effect on the maximum attainable value of H_X. Hence the gyros can effectively be driven to X axis saturation. From Table 1 the minimum ΔH_X available for a maneuver equals (6000 - 865) = 5135.

Let us conservatively assume that a minimum ΔH_X of 4500 lb-ft-sec is available for X axis maneuvers of either vehicle.

To achieve ΔH_X = 4500, the gyro gimbals rotate through approximately 90°. With a maximum design gimbal rate of 4.54°/sec., 20 seconds each are required for acceleration and deceleration. During this time the decoupled vehicle rotates 10.5° and the cluster rotates 7.5°.

The ΔH_X of 4500 produces vehicle rates of 0.62°/sec for the decoupled vehicle and 0.44°/sec for the cluster. With a maximum maneuver angle of 90°, the remaining 69° for the decoupled vehicle requires 110 seconds and the remaining 75° for the cluster requires 170 seconds.
Adding the times required for acceleration, constant rate rotation, and deceleration results in maximum maneuver times of 2.5 minutes for the decoupled vehicle and 3.5 minutes for the cluster. Since negligible dumping occurs during maneuvering, the maneuver time slightly reduces the dump capability.

Figure 9 shows the effect of these maneuver times on the undumped momentum per orbit. For $|\beta|$ less than approximately 30°, all the momentum accumulated during daylight pointing can be g.g. dumped. As 69.6° is approached, the pointing momentum increases rapidly, whereas the dumping interval decreases rapidly, giving rise to steep undumped momentum curves.

REACTION THRUST DUMP PROPELLANT REQUIREMENTS

Figure 10 shows the SM reaction thrust propellant required per orbit to cancel the undumped g.g. momentum. The dotted curves show the propellant required if g.g. dumping were not employed. Maneuver times are as calculated above.
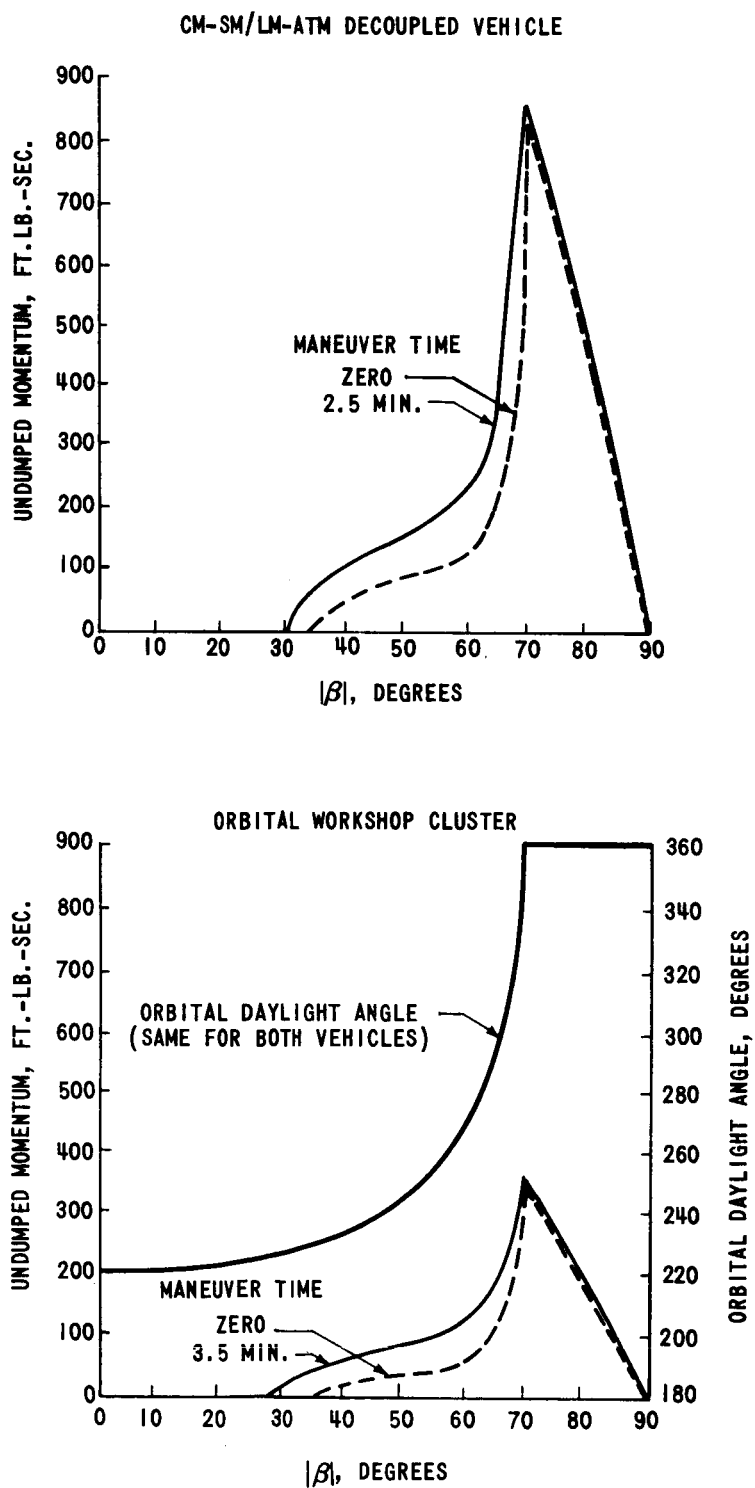
CM-SM/LM-ATM DECOUPLED VEHICLE

MANEUVER TIME
ZERO
2.5 MIN.

UNDUMPED MOMENTUM, FT.-LB.-SEC.

|β|, DEGREES

ORBITAL WORKSHOP CLUSTER

ORBITAL DAYLIGHT ANGLE
(SAME FOR BOTH VEHICLES)

MANEUVER TIME
ZERO
3.5 MIN.

UNDUMPED MOMENTUM, FT.-LB.-SEC.

ORBITAL DAYLIGHT ANGLE, DEGREES

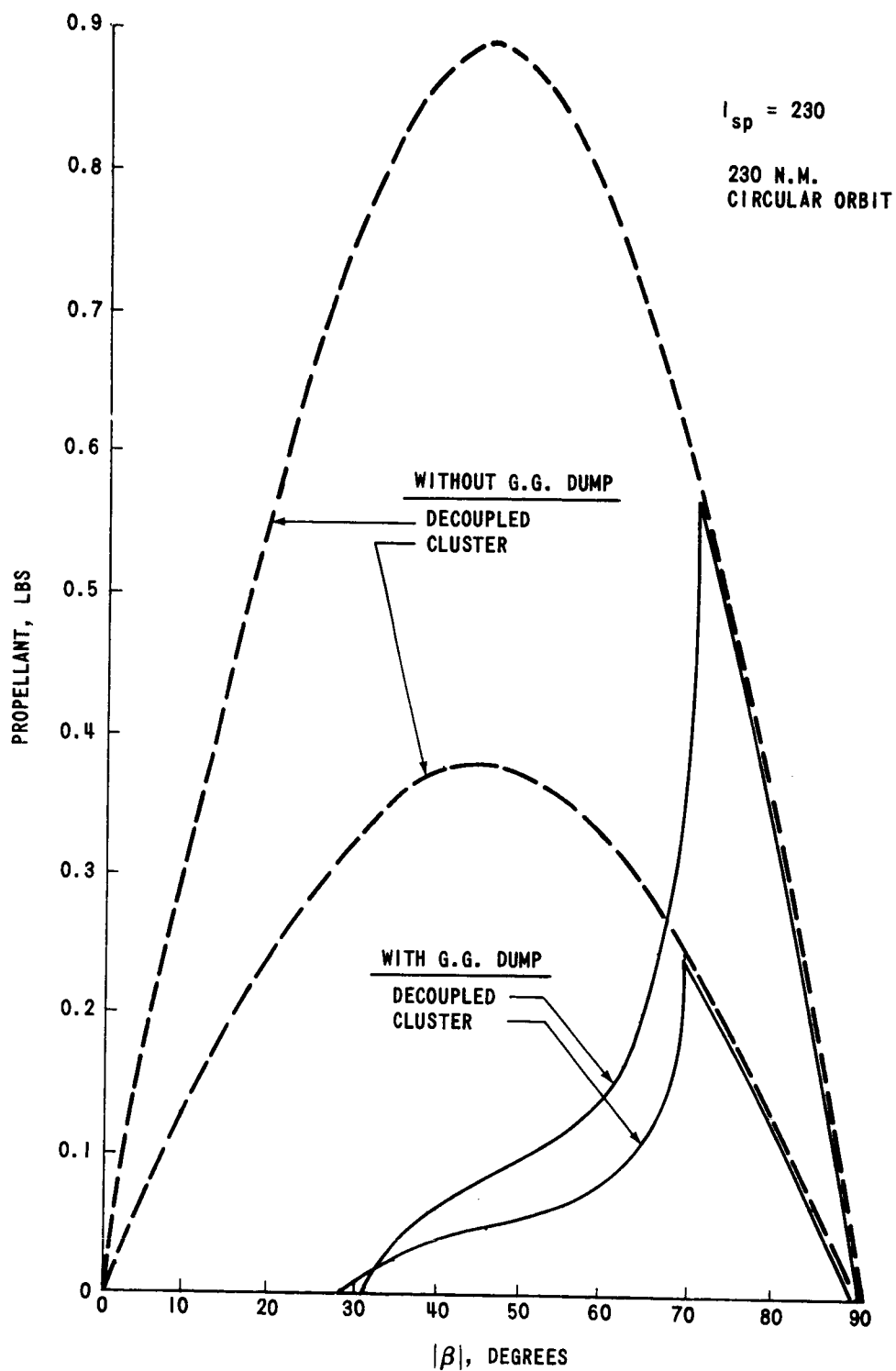|β|, DEGREES

FIGURE 9 - UNDUMPED MOMENTUM PER ORBIT

FIGURE IO - SM REACTION THRUST DUMP PROPELLANT REQUIRED PER ORBIT

The initial value of β for a mission depends upon the time of launch and the orbit inclination. After launch, β varies slowly due to orbital regression and the earth's rotation about the sun. The cluster, with a 28.5° orbital inclination can have a β varying approximately between ±52°. The CM-SM/LM-ATM decoupled mission, possibly with an inclination of 50°, can have a β varying between ±73°. The range actually encompassed by a mission depends upon the launch date and mission duration, and of course affects the total reaction thrust dump propellant required per mission. A simulation shows that without g.g. dumping, up to 340 lbs of dump propellant are required for a 28-day decoupled vehicle mission, depending upon launch date. When supplemented by g.g. dumping, 75 lbs are required.

Similarly, g.g. dumping reduces the dump propellant requirement for a 56-day cluster mission from 286 lbs to 20 lbs.
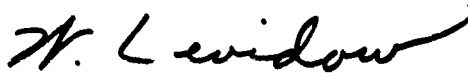
CONCLUSIONS

A single rotation, two maneuver gravity-gradient dump procedure can be used for the CM-SM/LM-ATM decoupled mission and also can be used as a manual backup for the orbital workshop cluster. Maneuvering is done during orbital darkness to maximize the solar viewing time.

For values of $|\beta|$ less than 30°, all the momentum accumulated during solar viewing can be dumped during orbital darkness. For $|\beta|$ greater than 30°, reaction thrust supplements g.g. dumping to prevent CMG momentum saturation.

A momentum management procedure has been suggested which results in maximum maneuver times of 2.5 minutes for the decoupled vehicle and 3.5 minutes for the cluster.

For a 28-day decoupled vehicle mission, this procedure reduces the maximum reaction thrust dump propellant requirement from 340 lbs to 75 lbs.

For a 56-day cluster mission, g.g. dumping reduces the propellant requirement from 286 lbs to 20 lbs.

W. Levidow

1022-WL-mef   W. Levidow

Attachment
  References

BELLCOMM, INC.

REFERENCES

1. J. Kranton, "Trip Report - Final Meeting of the Ad Hoc
    Committee on the ATM Pointing Control System (PCS), MSC
    on June 11, 1968 - Case 620", June 19, 1968.

2. B. D. Elrod, "Solar Pointing Variations in Earth Orbit and
    the Impact on Mission Design", Bellcomm Technical Report,
    in preparation.